ASSESSMENT OF THE MINIMUM DURATION FOR THE SPACECRAFT ATTITUDE MANEUVER

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Abstract

The problem of evaluation of time spent for attitude maneuver (rotation) during a finite rotation given a boundedness of angular rates and accelerations is solved. (The finite rotation means that the spacecraft body basis is moved from its initial position to the final one by the single rotation about an axis of finite rotation.) It is supposed that the maximum angular rates and accelerations about each spacecraft body axis are specified and different for each spacecraft body axis.

Very simple solution for the attitude maneuver minimum duration along with maximum angular rate and acceleration is suggested. The numerical results can be found by some uncomplicated procedure with several formulas and two tables.

Assessment of the minimum duration for the spacecraft attitude maneuver is most important for the rotations during the docking and undocking processes and prior to orbital correction.

Introduction

While calculating the spacecraft attitude maneuver (rotation), the problem of evaluation of time spent for rotation often arises because it is required usually to perform the rotation with minimum possible duration.

However, a boundedness of the angular rates and accelerations impedes the rapid implementation of attitude maneuver. Maximum absolute magnitudes of these rates and accelerations depend on spacecraft moments of inertia and available torque created by spacecraft actuators.

It is shown below how to find the attitude maneuver minimum duration along with maximum angular rate and acceleration if their components about each spacecraft body axis are bounded, and maneuver is performed as a finite rotation. The latter means that the spacecraft body basis is moved from its initial position to the final one by a single rotation about an axis of finite rotation[1,2].

Geometry of finite rotation is shown in Figure 1. Here, $\bar{e}$ is an axis of a finite rotation, χ is an angle of finite rotation, O is the spacecraft center of mass, X_I and X_F are the initial and final positions of the spacecraft body axis X, and α is an angle between the body axis X and axis of the finite rotation $\bar{e}$. This angle is constant during the rotation. The body axis X moves over the surface of the cone with vertex in the point O and cone angle 2α. The angles β and γ between two other spacecraft body axes (Y and Z) and axis of finite rotation $\bar{e}$ are also constant, and these axes move over the surface of the similar cones with the cone angles 2β and 2γ respectively.

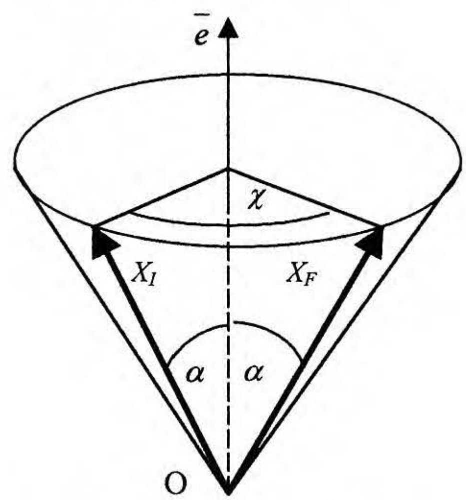

Fig. 1. Geometry of finite rotation

Whole information about a finite rotation except the angular rates and angular accelerations is included in components of the rotation quaternion $\bar{q}$ [1,2]

$$\left.\begin{aligned} q_0 = \cos\frac{\chi}{2},\ q_1 = \cos\alpha\sin\frac{\chi}{2}, \\ q_2 = \cos\beta\sin\frac{\chi}{2},\ q_3 = \cos\gamma\sin\frac{\chi}{2} \end{aligned}\right\} \quad (1)$$

Set up of Problem

For a given attitude maneuver, the angular rate and acceleration vectors

$$\bar{\omega} = \begin{Vmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{Vmatrix} \quad \text{and} \quad \bar{\varepsilon} = \begin{Vmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \end{Vmatrix} \tag{2}$$

coincide with the axis of finite rotation $\bar{e}$. The maximum allowable magnitude of these vectors (ω_{max}, and ε_{max}) depends on absolute value of the angular rate and acceleration limit for each body axis (ω_{Xmax}, ω_{Ymax}, ω_{Zmax} and ε_{Xmax}, ε_{Ymax}, ε_{Zmax}) and does not depend on their sign. That is why, it is enough to consider only positive value of these limits and the unit vector of finite rotation in the spacecraft body reference frame is

$$\bar{e}^0 = \begin{Vmatrix} |\cos\alpha| \\ |\cos\beta| \\ |\cos\gamma| \end{Vmatrix} \tag{3}$$

The limits mentioned above form some envelopes for the maximum angular rates and maximum angular accelerations in the first quadrant of the body reference frame (*X*, *Y*, *Z*). One of the mentioned envelopes (for angular rates) is shown in Figure 2. It is a parallelepiped with edges $|\omega_{Xmax}|$, $|\omega_{Ymax}|$, $|\omega_{Zmax}|$. The value of ω_{max} depends on which one of the parallelepiped faces (1, 2, and 3 in the Figure 2) is pierced by vector $\bar{e}$, i. e. on the components of the unit vector $\bar{e}^0$. In the case that is shown in the Figure 2, vector $\bar{e}$ pierces the face 2. That means that

$$\omega_Y = \omega_{Ymax} \quad \text{and} \quad \omega_{max} = \frac{|\omega_{Y\max}|}{|\cos\beta|} \tag{4}$$

Maximum Angular Rate and Acceleration

The determination of the pierced face number can be made through the use of the projection of the unit vector $\bar{e}^0$ on the coordinate planes *XY*, *YZ*, and *ZX*. The boards of the envelope on the mentioned coordinate planes are the rectangles, which sides are $|\omega_{Xmax}|$, $|\omega_{Ymax}|$, $|\omega_{Zmax}|$. Comparison between positions of the unit vector $\bar{e}^0$ projection and rectangle diagonal allows the determination of the number of parallelepiped faces

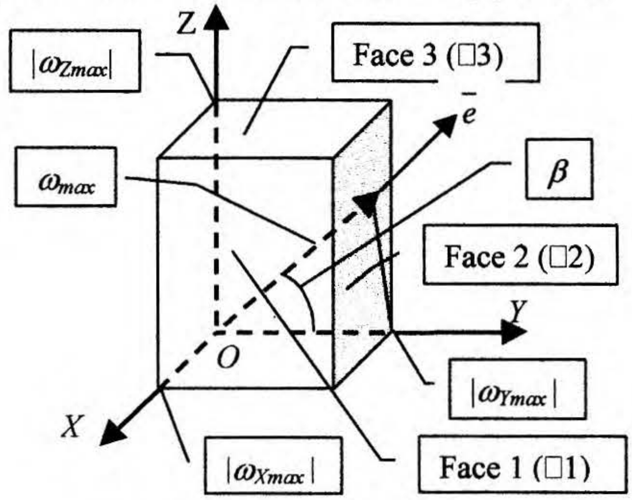

Figure 2. Envelope of maximum angular rate

(1, 2, 3), which can be pierced by vector $\bar{e}$. The position of the mentioned projection relative to the rectangle diagonal depends on relation between following ratios

$$\left.\begin{array}{l} \dfrac{|\cos\beta|}{|\cos\alpha|}, \dfrac{|\cos\alpha|}{|\cos\gamma|}, \dfrac{|\cos\gamma|}{|\cos\beta|}, \\[2ex] \dfrac{|\omega_{Y\max}|}{|\omega_{X\max}|}, \dfrac{|\omega_{X\max}|}{|\omega_{Z\max}|}, \dfrac{|\omega_{Z\max}|}{|\omega_{Y\max}|}. \end{array}\right\} \tag{5}$$

The situation when a projection of the unit vector $\bar{e}^0$ on the plane *XY* is closer to the *X* axis then diagonal of the rectangle with sides $|\omega_{Xmax}|$ and $|\omega_{Ymax}|$, is shown in Figure 3. In this case, $\dfrac{|\cos\beta|}{|\cos\alpha|} \le \dfrac{|\omega_{Y\max}|}{|\omega_{X\max}|}$, and vector $\bar{e}$ can pierce the half of the face 3 (shaded triangle) and whole face 1 (see Figure 3). In the opposite situation, when $\dfrac{|\cos\beta|}{|\cos\alpha|} > \dfrac{|\omega_{Y\max}|}{|\omega_{X\max}|}$, vector $\bar{e}$ can pierce another half of the face 3 (triangle without shading) and the whole face 2. The similar situations for planes *YZ* and *ZX* are shown in Figures 4 and 5. Two other cases are shown in Figure 4. In the first of them, $\dfrac{|\cos\gamma|}{|\cos\beta|} \le$

$\frac{|\omega_{Z\max}|}{|\omega_{Y\max}|}$, and the vector $\bar{e}$ can pierce the shaded half (triangle) of the face 1 and whole face 2.

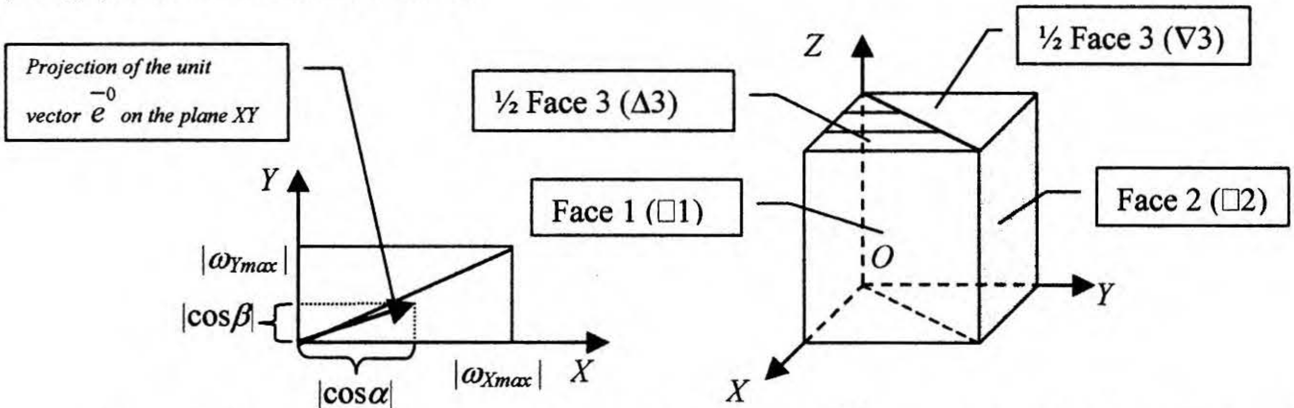

Figure 3. Correlation between position of the unit vector $\bar{e}^0$ projection on the plane XY and faces pierced by vector $\bar{e}$

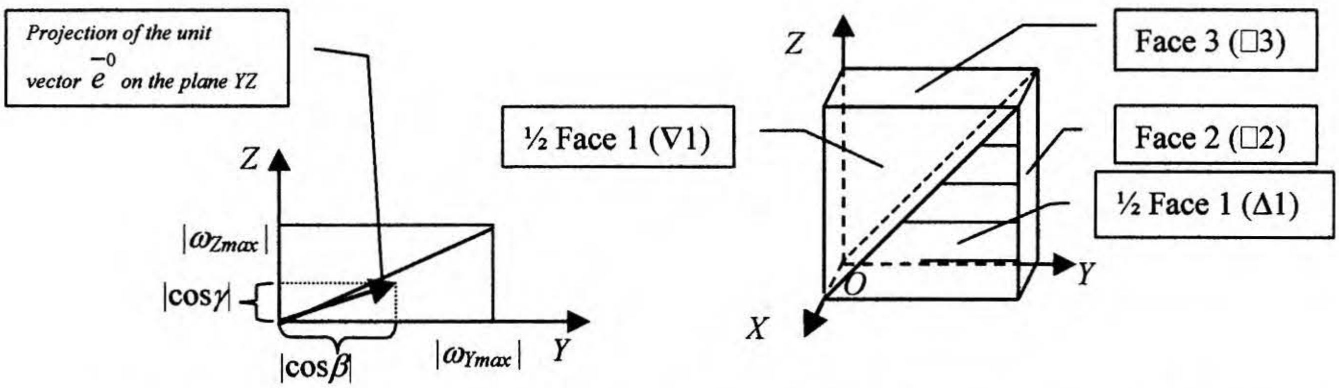

Figure 4. Correlation between position of the unit vector $\bar{e}^0$ projection on the plane YZ and faces pierced by vector $\bar{e}$

In the opposite situation, when $\frac{|\cos\gamma|}{|\cos\beta|} > \frac{|\omega_{Z\max}|}{|\omega_{Y\max}|}$, vector $\bar{e}$ can pierce another half of the face 1 and whole face 3. Also, two cases are shown in Figure 5. When $\frac{|\cos\alpha|}{|\cos\gamma|} \le \frac{|\omega_{X\max}|}{|\omega_{Z\max}|}$, vector $\bar{e}$ can pierce the shaded half of the face 2 and whole face 3. When $\frac{|\cos\alpha|}{|\cos\gamma|} > \frac{|\omega_{X\max}|}{|\omega_{Z\max}|}$, vector $\bar{e}$ can pierce another half of the face 2 and whole face 1.

The total quantity of the possible combinations of the inequalities, that establish relation between the ratios listed in (5), is eight.

The results obtained from Figures 2-5 for angular rates are summarized in the Table 1. This table covers all eight options for relations between ratios listed in (5) and includes number of face (rectangle) or part of face (triangle), which is pierced by vector $\bar{e}$ according to these relations. The mentioned rectangles and triangles are shown in the bottom line for each option in the columns 3, 4, and 5 of the Table. These rectangles and triangles are actually some sets. Intersection of these sets determinates what component of the vector $\bar{\omega}$ is restricted.

Table 1

Option	*Relation between ratios* $\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert}, \frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert}, \frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert}$ *and* $\frac{\lvert\omega_{Y\max}\rvert}{\lvert\omega_{X\max}\rvert}, \frac{\lvert\omega_{X\max}\rvert}{\lvert\omega_{Z\max}\rvert}, \frac{\lvert\omega_{Z\max}\rvert}{\lvert\omega_{Y\max}\rvert}$			*Face part number*	ω_{max}
1	2	3	4	5	6
1	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} \le \frac{\lvert\omega_{Y\max}\rvert}{\lvert\omega_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} > \frac{\lvert\omega_{X\max}\rvert}{\lvert\omega_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} \le \frac{\lvert\omega_{Z\max}\rvert}{\lvert\omega_{Y\max}\rvert}$	Δ1	$\omega_{max} = \frac{\lvert\omega_{X\max}\rvert}{\lvert\cos\alpha\rvert}$
	□1+Δ3	□1+∇2	□2+Δ1		
2	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} \le \frac{\lvert\omega_{Y\max}\rvert}{\lvert\omega_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} > \frac{\lvert\omega_{X\max}\rvert}{\lvert\omega_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} > \frac{\lvert\omega_{Z\max}\rvert}{\lvert\omega_{Y\max}\rvert}$	∇1	
	□1+Δ3	□1+∇2	□3+∇1		
3	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} > \frac{\lvert\omega_{Y\max}\rvert}{\lvert\omega_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} \le \frac{\lvert\omega_{X\max}\rvert}{\lvert\omega_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} \le \frac{\lvert\omega_{Z\max}\rvert}{\lvert\omega_{Y\max}\rvert}$	Δ2	$\omega_{max} = \frac{\lvert\omega_{Y\max}\rvert}{\lvert\cos\beta\rvert}$
	□2+∇3	□3+Δ2	□2+Δ1		
4	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} > \frac{\lvert\omega_{Y\max}\rvert}{\lvert\omega_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} > \frac{\lvert\omega_{X\max}\rvert}{\lvert\omega_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} \le \frac{\lvert\omega_{Z\max}\rvert}{\lvert\omega_{Y\max}\rvert}$	∇2	
	□2+∇3	□1+∇2	□2+Δ1		
5	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} \le \frac{\lvert\omega_{Y\max}\rvert}{\lvert\omega_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} \le \frac{\lvert\omega_{X\max}\rvert}{\lvert\omega_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} > \frac{\lvert\omega_{Z\max}\rvert}{\lvert\omega_{Y\max}\rvert}$	Δ3	$\omega_{max} = \frac{\lvert\omega_{Z\max}\rvert}{\lvert\cos\gamma\rvert}$
	□1+Δ3	□3+Δ2	*□3+∇1*		
6	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} > \frac{\lvert\omega_{Y\max}\rvert}{\lvert\omega_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} \le \frac{\lvert\omega_{X\max}\rvert}{\lvert\omega_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} > \frac{\lvert\omega_{Z\max}\rvert}{\lvert\omega_{Y\max}\rvert}$	∇3	
	□2+∇3	□3+Δ2	□3+∇1		
7	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} \le \frac{\lvert\omega_{Y\max}\rvert}{\lvert\omega_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} \le \frac{\lvert\omega_{X\max}\rvert}{\lvert\omega_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} \le \frac{\lvert\omega_{Z\max}\rvert}{\lvert\omega_{Y\max}\rvert}$	*Main diagonal Ok (see Fig. 6)*	$\omega_{max} = \frac{\lvert\omega_{X\max}\rvert}{\lvert\cos\alpha\rvert} = \frac{\lvert\omega_{Y\max}\rvert}{\lvert\cos\beta\rvert} = \frac{\lvert\omega_{Z\max}\rvert}{\lvert\cos\gamma\rvert}$
	□1+Δ3	□3+Δ2	□2+Δ1		
8	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} > \frac{\lvert\omega_{Y\max}\rvert}{\lvert\omega_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} > \frac{\lvert\omega_{X\max}\rvert}{\lvert\omega_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} > \frac{\lvert\omega_{Z\max}\rvert}{\lvert\omega_{Y\max}\rvert}$		
	□2+∇3	□1+∇2	□3+∇1		

For example, ∇1 in column 5 for option 2 (triangle without shading on the face1) is the intersection of the □1 in column 2, □1 in column 3, and ∇1 in column 4. In the same way, Δ1 in the column 5 for option 1 (shaded triangle on the face 1) is the

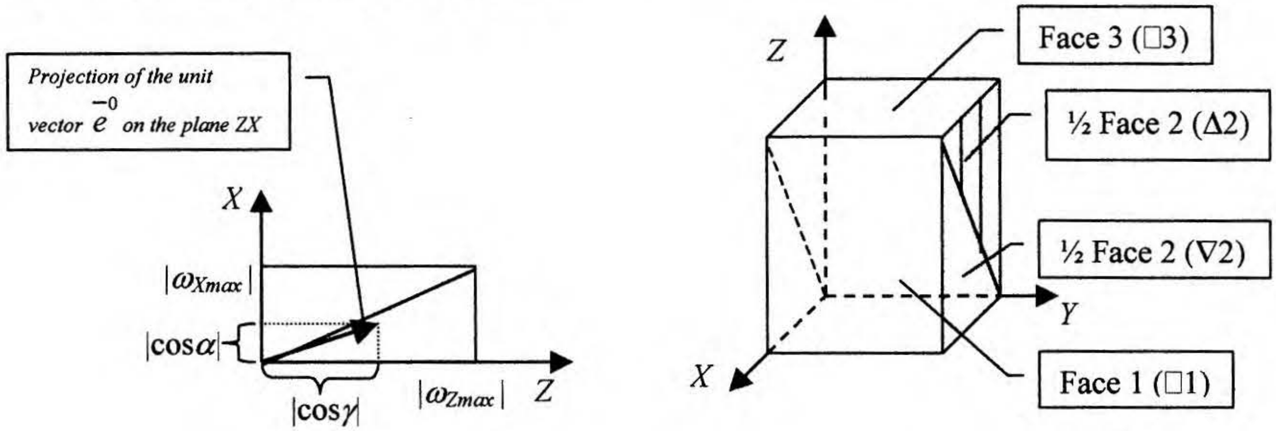

Figure 5. Correlation between position of the unit vector $\bar{e}^0$ projection on the plane ZX and faces pierced by vector $\bar{e}$

intersection of the □1 in column 2, □1 in column 3, and Δ1 in column 4. So, options 1 and 2 together form the face 1, and vector $\bar{e}$ pierces face 1. In this case ω_X is the restricted component, and $\omega_{max} = \dfrac{|\omega_{X\max}|}{|\cos\alpha|}$ (column 6). Options 3 and 4 together form the face 2, and in this case ω_Y is the

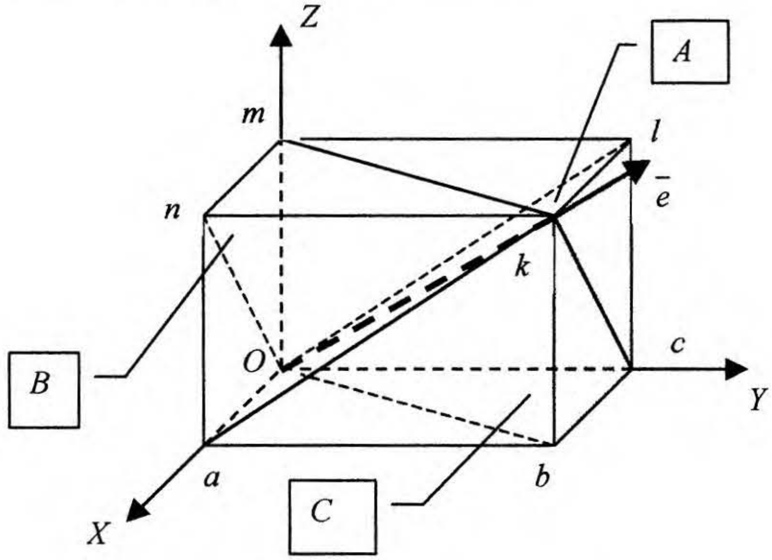

Figure 6. Vector $\bar{e}$ for options 7 and 8

restricted component, and $\omega_{max} = \dfrac{|\omega_{Y\max}|}{|\cos\beta|}$. Options 5 and 6 together form the face 3, and in this case ω_Z is the restricted component, and $\omega_{max} = \dfrac{|\omega_{Z\max}|}{|\cos\gamma|}$. Options 7 and 8 are different from other options because it seems that their sets (for example, sets □2+∇3, □1+∇2, and □3+∇1 for option 8) do not have any intersections. In fact the mentioned sets have the intersections that are the boundaries of these sets. It is clear from Figure 6. The planes *A*, *B*, and *C* on this Figure include axes *OX*, *OY*, *OZ* and diagonals *Ol*, *On*, and *Ob* of the rectangles *Oclm*, *Oanm*, and *Oabc* respectively. Vector $\bar{e}$ coincides with these planes when inequalities in lines 7 and 8 of the Table 1 change into equalities. In other words, these planes are the boundary sets of possible position of the vector $\bar{e}$, and their intersection is the main diagonal Ok of the parallelepiped. So, in this case, vector $\bar{e}$ coincides with the parallelepiped main diagonal, and

$$\left.\begin{aligned} \omega_{\max} &= \sqrt{\omega_{X\max}^2 + \omega_{Y\max}^2 + \omega_{Z\max}^2} \\ \omega_{\max} &= \frac{|\omega_{X\max}|}{|\cos\alpha|} = \frac{|\omega_{Y\max}|}{|\cos\beta|} = \frac{|\omega_{Z\max}|}{|\cos\gamma|} \end{aligned}\right\} \quad (6)$$

The column 6 in the Table 1 includes the formulas for calculation the ω_{max} for all possible relations between ratios (5).

The envelope for angular acceleration is also a parallelepiped like for angular rate. Its edges are $|\varepsilon_{Xmax}|$, $|\varepsilon_{Ymax}|$, $|\varepsilon_{Zmax}|$. The value of ε_{max} depends on which one of the faces (1, 2, and 3) of this parallelepiped is pierced by vector $\bar{e}$ like for ω_{max}. The same type of analysis, which was performed for (5), can be completed for the ratios

$$\left.\begin{aligned} &\frac{|\cos\beta|}{|\cos\alpha|}, \frac{|\cos\alpha|}{|\cos\gamma|}, \frac{|\cos\gamma|}{|\cos\beta|}, \\ &\frac{|\varepsilon_{Y\max}|}{|\varepsilon_{X\max}|}, \frac{|\varepsilon_{X\max}|}{|\varepsilon_{Z\max}|}, \frac{|\varepsilon_{Z\max}|}{|\varepsilon_{Y\max}|}. \end{aligned}\right\} \quad (7)$$

For the ratios listed in (7), the same eight combinations of the inequalities exist, and the Table 2, which is similar to Table 1, can be built like for ratios (5). The Table 2 (see below) can be obtained from the Table 1 by substitution $|\varepsilon_{Xmax}|$, $|\varepsilon_{Ymax}|$, $|\varepsilon_{Zmax}|$ and ε_{max} instead of $|\omega_{Xmax}|$, $|\omega_{Ymax}|$, $|\omega_{Zmax}|$ and ω_{max}. Column 5 in Table 2 includes the formulas to calculate ε_{max} for all possible relations between ratios (7).

Minimum Duration of Attitude Maneuver

After determination ω_{max} and ε_{max}, the minimum duration of an attitude maneuver can be assessed. The time plot for the angular rate ω is shown in the Figure 7. First, the angular rate ω increases from

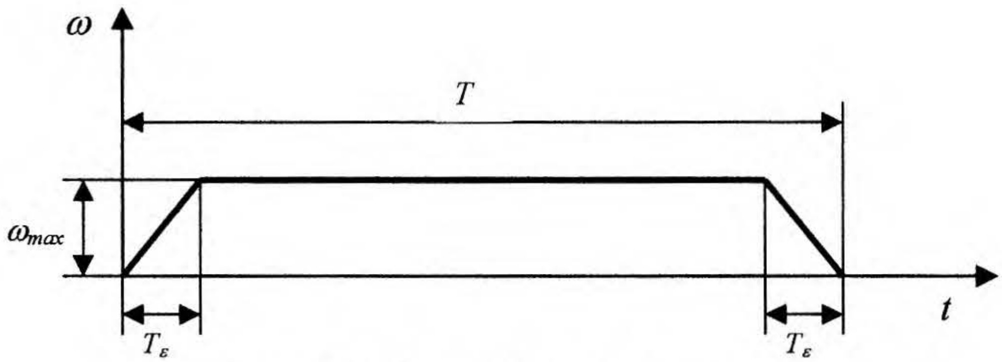

Figure 7. Angular rate for an attitude maneuver

0 to ω_{max} with the angular acceleration ε_{max} at the interval T_ε. Then, the angular rate ω is constant and equal to ω_{max} at the interval T-$2T_\varepsilon$ and after that decreases from ω_{max}. to 0 with the angular acceleration (-ε_{max}) at the interval T_ε. Obviously, the time plot that is shown in the Figure 7 provided a minimum duration of rotation. If T (see Figure 7) is a duration of attitude maneuver and χ is an angle of finite rotation, then formulas

$$\chi = \int_0^T \omega dt = \omega_{\max}(T - 2T_\varepsilon) + 2\varepsilon_{\max}\frac{T_\varepsilon^2}{2} \quad (8)$$

and

$$\omega_{\max} = \varepsilon_{\max} T_\varepsilon \quad (9)$$

are true.

Table 2

Option	***Relation between ratios*** $\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert}, \frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert}, \frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert}$ ***and*** $\frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\varepsilon_{X\max}\rvert}, \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\varepsilon_{Z\max}\rvert}, \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\varepsilon_{Y\max}\rvert}$			ε_{max}
1	2	3	4	5
1	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} \le \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\varepsilon_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} > \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\varepsilon_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} \le \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\varepsilon_{Y\max}\rvert}$	$\varepsilon_{max} = \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\cos\alpha\rvert}$
2	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} \le \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\varepsilon_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} > \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\varepsilon_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} > \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\varepsilon_{Y\max}\rvert}$	
3	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} > \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\varepsilon_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} \le \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\varepsilon_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} \le \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\varepsilon_{Y\max}\rvert}$	$\varepsilon_{max} = \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\cos\beta\rvert}$
4	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} > \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\varepsilon_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} > \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\varepsilon_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} \le \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\varepsilon_{Y\max}\rvert}$	
5	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} \le \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\varepsilon_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} \le \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\varepsilon_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} > \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\varepsilon_{Y\max}\rvert}$	$\varepsilon_{max} = \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\cos\gamma\rvert}$
6	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} > \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\varepsilon_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} \le \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\varepsilon_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} > \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\varepsilon_{Y\max}\rvert}$	
7	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} \le \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\varepsilon_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} \le \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\varepsilon_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} \le \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\varepsilon_{Y\max}\rvert}$	$\varepsilon_{max} = \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\cos\alpha\rvert} = \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\cos\beta\rvert} = \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\cos\gamma\rvert}$
8	$\frac{\lvert\cos\beta\rvert}{\lvert\cos\alpha\rvert} > \frac{\lvert\varepsilon_{Y\max}\rvert}{\lvert\varepsilon_{X\max}\rvert}$	$\frac{\lvert\cos\alpha\rvert}{\lvert\cos\gamma\rvert} > \frac{\lvert\varepsilon_{X\max}\rvert}{\lvert\varepsilon_{Z\max}\rvert}$	$\frac{\lvert\cos\gamma\rvert}{\lvert\cos\beta\rvert} > \frac{\lvert\varepsilon_{Z\max}\rvert}{\lvert\varepsilon_{Y\max}\rvert}$	

The final formula for the minimum duration of attitude maneuver T that can be obtained from formulas (8) and (9) is

$$T = \frac{\chi}{\omega_{\max}} + \frac{\omega_{\max}}{\varepsilon_{\max}} \tag{10}$$

Calculation Procedure

The minimum duration of the spacecraft attitude maneuver (finite rotation) can be calculated by the following procedure:

1. Calculation of the angle of the finite rotation χ and the direction cosines cosα, cosβ, cosγ of the finite rotation axis $\bar{e}$ in the spacecraft body reference frame (X, Y, Z) using formulas (1) for the components of the of the rotation quaternion $\bar{q}$:

$$\left.\begin{aligned} \chi = 2\arccos q_0, \quad \cos\alpha = \frac{q_1}{\sqrt{1-q_0^2}}, \\ \cos\beta = \frac{q_2}{\sqrt{1-q_0^2}}, \quad \cos\gamma = \frac{q_3}{\sqrt{1-q_0^2}}. \end{aligned}\right\} \tag{11}$$

2. Selection of the ω_{max} from the Table 1 using ratios (5).

3. Selection of the ε_{max} from the Table 2 using ratios (7).

4. Calculation of the minimum duration of the spacecraft attitude maneuver T using (10).

References

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